CASCADE MECHANISM OF NONLINEAR INTERACTIONS BETWEEN MODES IN A TURBULENT PLASMA

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SHORT ABSTRACT

(For Information Retrieval)

A hydrodynamic model of plasma turbulence is developed. The cascade concept is utilized for the problem of closure. The spectra of kinetic and electrostatic energies are investigated for both the collisional and collisionless cases. The anomalous diffusion, called Bohm diffusion, is also derived, and found to play an important role in the nonlinear transport properties of the problem.

1. INTRODUCTION

There are several areas of marked similarity between the nonlinear wave interactions in electron-ion gas plasmas and in electron-hole solid plasmas. Such a similarity in turbulence has been demonstrated by Betsy Ancker-Johnson[1] in solid plasmas by an experiment equivalent to that in gas plasmas, and has been stressed by Hoh[2] on theoretical considerations. Theories[3] on turbulence in gas and solid plasmas have been very sketchy, and up to date, only some phenomenological and quasilinear considerations have been treated, valid for a weak turbulence. In the following, we attempt a hydrodynamical model of strong turbulence in the aim of analyzing the nonlinear interactions across the individual spectra and between the spectra of kinetic and electrostatic energies. In view of the similarity in turbulence between the solid and gas plasmas, as mentioned above, we shall retain the notations of gas plasmas in our theoretical development, and shall not attempt to discriminate the singular distinctions of solid plasmas.

2. CASCADE SYSTEM

We consider a quasineutral plasma consisting of ions and electrons. The ions have a velocity $\underset{\sim}{u}$, a charge e and a mass M. A magnetic field of constant strength 0,0,B, has a cyclotron frequency $\omega_c = eB/M$, and is assumed large compared to the pressure force. The electrons are hot, and assume a Boltzmann distribution

$$n = n_0 \exp(\psi/a)$$

for the number density n for electrons or ions, n_0 is the average density, a is the phase velocity

$$a = (KT_e/M)^{\frac{1}{2}}$$

with an electron temperature T_e and the Boltzmann constant K. Further ψ is the product of the self-consistent electric potential by e/M. Write the Navier-Stokes equation of motion and the equation of continuity for the fluctuations transverse to the magnetic field:

$$\frac{du_i}{dt} - \varepsilon_{ijk}\,\gamma_k\,\omega_c\,u_j = -a\frac{\partial\psi}{\partial x_i} + \nu\nabla^2 u_i \qquad (1a)$$

$$\frac{d\psi}{dt} = -a\,\nabla\cdot\underset{\sim}{u}\,, \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + \underset{\sim}{u}\cdot\nabla \qquad (1b)$$

Here ν is the kinematic viscosity, ε is an antisymmetric unit tensor, and γ = 0,0,1.

Using the cascade decomposition

$$\underset{\sim}{u} = \underset{\sim}{u}^{\circ} + \underset{\sim}{u}', \qquad \psi = \psi^{\circ} + \psi' \qquad (2)$$

where

$$\underset{\sim}{u}(\underset{\sim}{x}) = \int_{-\infty}^{\infty} d\underset{\sim}{k}\; e^{i\underset{\sim}{k}\underset{\sim}{x}}\,\underset{\sim}{u}(\underset{\sim}{k})$$

$$= \int_{|k|=0}^{k} d\underset{\sim}{k}\; e^{i\underset{\sim}{k}\underset{\sim}{x}}\,\underset{\sim}{u}(\underset{\sim}{k}) + \int_{k}^{\infty} d\underset{\sim}{k}\; e^{i\underset{\sim}{k}\underset{\sim}{x}}\,\underset{\sim}{u}(\underset{\sim}{k})$$

and

$$\underset{\sim}{u}^{\circ}(\underset{\sim}{x}) = \int_{|k|=0}^{k} d\underset{\sim}{k}\; e^{i\underset{\sim}{k}\underset{\sim}{x}}\,\underset{\sim}{u}(\underset{\sim}{k})\,, \qquad \underset{\sim}{u}'(\underset{\sim}{x}) = \int_{k}^{\infty} d\underset{\sim}{k}\; e^{i\underset{\sim}{k}\underset{\sim}{x}}\,\underset{\sim}{u}(\underset{\sim}{k})$$

Similarly for ψ, ψ° and ψ', we find the following cascade system:

$$\frac{Du_i^{\circ}}{Dt} - \varepsilon_{ijk}\,\gamma_k\,\omega_c\,u_j^{\circ} = E_i^{\circ} - \left\langle u_j'\frac{\partial u_i'}{\partial x_j}\right\rangle + \nu\nabla^2 u_i^{\circ} \qquad (3a)$$

$$\frac{D\psi^{\circ}}{Dt} = -a\,\nabla\cdot\underset{\sim}{u}^{\circ} - \langle\underset{\sim}{u}'\cdot\nabla\psi'\rangle \qquad (3b)$$

$$\frac{du_i'}{dt} - \varepsilon_{ijk}\,\gamma_k\,\omega_c\,u_j' - \nu\nabla^2 u_i' = E_i' - u_j'\frac{\partial u_i^{\circ}}{\partial x_j} \qquad (4a)$$

$$\frac{d\psi'}{dt} = -a\,\nabla\cdot\underset{\sim}{u}' - u'\cdot\nabla\psi^{\circ} \qquad (4b)$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \underset{\sim}{u}^{\circ}\cdot\nabla$$

where

$$\underset{\sim}{E}^{\circ} = -a\nabla\psi^{\circ} \qquad \text{and} \qquad \underset{\sim}{E}' = -a\nabla\psi'$$

represent the self-consistent electric fields multiplied by e/M.

The variables $\underset{\sim}{u}^{\circ}$ and ψ° are called the large scale fluctuations, and the variables $\underset{\sim}{u}'$ and ψ' are called the small scale fluctuations.

The system of equations (3) and (4) has been derived from (1), upon substituting for (2) and on the basis of the following assumptions:

(i) The variables $\underset{\sim}{u}^{\circ}$ and ψ° vary slowly and the variables $\underset{\sim}{u}'$ and ψ' vary rapidly, so that a variable transition scale k can be used to distinguish between the parts by a <u>local</u> average, denoted by $\langle\cdots\rangle$. A <u>general</u> average over an infinite length will be denoted by a bar. This is the assumption known as the quasi-stationarity assumption.

(ii) The turbulent motions are isotropic in the plane perpendicular to the magnetic field. The quantities $\underset{\sim}{u}$, ψ, $\underset{\sim}{u}^{\circ}$ and ψ° are homogeneous generally, but the quantities u', ψ' are only locally homogeneous, i.e., within an interval k^{-1}, as their averages have been so defined.

The equations of kinetic and potential energies are derived by multiplying (3a) and (3b) by u_i° and ψ° respectively, by averaging and using the assumptions (i) and (ii):

$$\frac{1}{2}\,\overline{\frac{D\underset{\sim}{u}^{\circ 2}}{Dt}} = -\left[\Phi_k + (\nu+\nu_k)\,R^{\circ}\right] \tag{5a}$$

$$\frac{1}{2}\,\overline{\frac{D\psi^{\circ 2}}{Dt}} = -\left[-\Phi_k + \lambda_k J^{\circ}\right] \tag{5b}$$

where

$$\Phi_k = -\,\overline{\underset{\sim}{u}^{\circ}\cdot\underset{\sim}{E}^{\circ}}$$

or identically

$$\Phi_k = \overline{\underset{\sim}{u}'\cdot\underset{\sim}{E}'}$$

as shown from averaging (3b). We have denoted

$$R^{\circ} = \overline{(\partial u_i^{\circ}/\partial x_j)^2}, \qquad J^{\circ} = \overline{(\nabla\psi^{\circ})^2}$$

$$R = R^{\circ}(k=\infty), \qquad J = J^{\circ}(k=\infty)$$

and written

$$\overline{\langle u'_j\, \partial u'_i / \partial x_j \rangle\, u^\circ_i} = \nu_k\, R^\circ \tag{7a}$$

$$\overline{\langle u' \cdot \nabla \psi' \rangle\, \psi^\circ} = \lambda_k\, T^\circ \tag{7b}$$

The expressions (6) and (7) are transport functions; more specifically Φ_k is a production function, transforming the kinetic energy into the potential energy in the form of electrostatic fluctuations. $\nu_k R^\circ$ and $\lambda_k T^\circ$ are kinetic and potential transfer functions, describing the transfer of energy across the kinetic and potential spectra respectively.

The equations (5) represent the evolution of the kinetic and potential energy spectra. In the universal range, i.e., in the range of not too small k , the rate of change of energy in the portion (0,k) of the spectrum is not very much different from that in the whole spectrum ($0, \infty$) which is a constant in equilibrium. Under such circumstances, we simplify (5) to

$$\Phi_k + (\nu + \nu_k)\, R^\circ = \Phi_\infty + \nu R \tag{8a}$$

$$-\Phi_k + \lambda_k\, T^\circ = -\Phi_\infty \tag{8b}$$

noting that

$$\nu_\infty = 0 \qquad \text{and} \qquad \lambda_\infty = 0$$

as, by definition, there is no net energy transfer in the whole spectrum.

3. TRANSPORT FUNCTIONS

The transfer functions (7) involve the statistical effects from the fluctuations $\underset{\sim}{u}'$ and ψ' which are calculated by integrating (4a). During this procedure, it is important to note that the functions (7) contain a coupling with $\underset{\sim}{u}^\circ$ or ψ° to its first power, and therefore the integration of (4a) should select terms of such a coupling only, for the general average to make a non zero contribution, all other terms having no contribution. Hence, we find the stresses in (3) and (7):

$$\langle u'_j\, \partial u'_i / \partial x_j \rangle = -\nu_k\, \nabla^2 u^\circ_i$$

$$\langle \underset{\sim}{u}' \cdot \nabla \psi' \rangle = -\lambda_k\, \nabla^2 \psi^\circ$$

yielding, by a general average,

$$\overline{u_i^\circ \langle u_j' \, \partial u_i'/\partial x_j \rangle} = -\nu_k \, \overline{u_i^\circ \, \nabla^2 u_i^\circ}$$

$$= \nu_k R^\circ$$

and

$$\overline{\psi^\circ \langle \underset{\sim}{u}' \cdot \nabla \psi' \rangle} = -\lambda_k \, \overline{\psi^\circ \nabla^2 \psi^\circ}$$

$$= \lambda_k J^\circ$$

The terms $\nabla^2 \overline{\underset{\sim}{u}^{\circ 2}}$ and $\nabla^2 \overline{\psi^{\circ 2}}$ have been dropped on account of the homogeneity assumption (ii). From the above operations, we find

$$\nu_k = \frac{1}{6} \int_{-\infty}^{\infty} d\tau \, \langle \underset{\sim}{u}'(0) \cdot \underset{\sim}{u}'(\tau) \rangle \cos \omega_c \tau \tag{10a}$$

$$\lambda_k = \frac{1}{6} \int_{-\infty}^{\infty} d\tau \, \langle \underset{\sim}{u}'(0) \cdot \underset{\sim}{u}'(\tau) \rangle \tag{10b}$$

In the derivation of (10a) from (4a), we have dropped the viscous effect.

(b) The production function (6b), on the other hand, does not involve a coupling with $\underset{\sim}{u}^\circ$ and ψ°. Therefore, the determination of $\underset{\sim}{u}'$ from (4a), for the purpose of evaluating (6b), should select terms without a coupling, as any term with a coupling with $\underset{\sim}{u}^\circ$ or ψ° to its first power would vanish by a general average. Hence we find the production function (6b) to be

$$\Phi_k = \frac{1}{6} \int_{-\infty}^{\infty} d\tau \, \langle \underset{\sim}{E}'(0) \cdot \underset{\sim}{E}'(\tau) \rangle \cos \omega_c \tau \tag{10c}$$

again we have dropped the viscous effect.

4. TRANSPORT COEFFICIENTS

The time integration of the auto-correlations of velocity and field fluctuations define the transport coefficients in (10). They are very similar.

Using the Fourier transform of a function truncated within a time interval T and a length interval X, with

$$\chi = \pi^4/TX^3$$

we can write

$$\overline{\underset{\sim}{u}(t,\underset{\sim}{x}) \cdot \underset{\sim}{u}(t',\underset{\sim}{x}')}^{\,t,x} = \chi \iint_{-\infty}^{\infty} d\omega \, d\underset{\sim}{k} \, |\underset{\sim}{u}(\omega,\underset{\sim}{k})|^2 \, e^{-i(\omega - \underset{\sim}{k}\underset{\sim}{u})\tau}$$

where the bar with superscripts (t,x) denotes an average over time and space within the above mentioned truncation intervals. Further $t - t' = \tau$, and $\underset{\sim}{x} - \underset{\sim}{x}' = \underset{\sim}{k}\underset{\sim}{u}\tau$, and the Lagangian time integration gives

$$\int_{-\infty}^{\infty} d\tau \ \overline{\underset{\sim}{u}(t,\underset{\sim}{x})\cdot \underset{\sim}{u}(t',\underset{\sim}{x}')}^{t,x} \cos\omega_c\tau$$

$$= \chi \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\underset{\sim}{k} \ |\underset{\sim}{u}(\omega,\underset{\sim}{k})|^2 \int_{-\infty}^{\infty} d\tau \ e^{-i(\omega-\underset{\sim}{k}\,\underset{\sim}{u})\tau} \cos\omega_c\tau$$

$$= \chi \int_{-\infty}^{\infty} d\omega \int_{0}^{\infty} dk \ 2\pi k^2 |\underset{\sim}{u}(\omega,k)|^2 \int_{-1}^{+1} d\mu \int_{-\infty}^{\infty} d\tau \ e^{-i(\omega-ku\mu)\tau} \cos\omega_c\tau$$

$$(\mu = \cos\theta)$$

$$\cong \frac{\pi\chi}{\omega_c} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\underset{\sim}{k} \ |\underset{\sim}{u}(\omega,\underset{\sim}{k})|^2$$

$$= \pi\omega_c^{-1} \ \overline{\underset{\sim}{u}^2}^{t,x}$$

Here we have introduced the approximation $k u \cong \omega_c$.

Similarly we conclude that

$$\int_{-\infty}^{\infty} d\tau \ \langle \underset{\sim}{u}'(0)\cdot \underset{\sim}{u}'(\tau)\rangle \cos\omega_c\tau = \pi\omega_c^{-1}\langle \underset{\sim}{u}'^2\rangle$$

$$\int_{-\infty}^{\infty} d\tau \ \langle \underset{\sim}{E}'(0)\cdot \underset{\sim}{E}'(\tau)\rangle \cos\omega_c\tau = \pi\omega_c^{-1}\langle \underset{\sim}{E}'^2\rangle$$

Hence (10a) and (10c) become

$$\nu_k = \frac{\pi}{6}\omega_c^{-1}\langle \underset{\sim}{u}'^2\rangle \tag{11a}$$

$$\Phi_k = \frac{\pi}{6}\omega_c^{-1}\langle \underset{\sim}{E}'^2\rangle \tag{11b}$$

Introduce the spectral functions F(k) and G(k) for the kinetic and potential energies respectively. We can write

$$\frac{1}{2}\overline{\underset{\sim}{u}^{o2}} = \int_0^k dk \, F, \qquad \frac{1}{2}\overline{\psi^{o2}} = \int_0^k dk \, G$$

$$R^\circ = 2\int_0^k dk \, k^2 F, \qquad J^\circ = 2\int_0^k dk \, k^2 G$$

and reduce (11a) to

$$\nu_k = \frac{\pi}{3}\omega_c^{-1}\int_k^{\infty} dk \, F \tag{12a}$$

Since

$$E' = -a\nabla\psi'$$

and

$$\langle E'^2\rangle = a^2\langle(\nabla\psi')^2\rangle$$

$$= a^2 \, 2\int_k^{\infty} dk \, k^2 G \equiv a^2(J - J^\circ)$$

we can rewrite (11b) in the form

$$\Phi_k = \lambda_B (J - J^\circ) \qquad (12b)$$

with

$$\lambda_B = \frac{\pi}{6} a^2 \omega_c^{-1} \qquad (12c)$$

the diffusion coefficient of Bohm. It is found to be proportional to ω_c^{-1}, in contrast to the classical diffusion across the magnetic field which is proportional to ω_c^{-2}. The formula (12c) has been proposed by Bohm without demonstration. His proposed formula contained an empirical numerical coefficient smaller than in (12c).

Formula (12b) determines

$$\Phi_{k=\infty} = 0 \qquad (12d)$$

which substitution into (8b) produces

$$\Phi_k = \lambda_k J^\circ \qquad (12e)$$

The evaluation of λ_k according to (10b) requires equally a Lagrangian time integration, with the parameter ω_c in (11a) replaced by a variable relaxation frequency ω^*, and we find

$$\int_{-\infty}^{\infty} d\tau \ \langle \underset{\sim}{u}'(0) \cdot \underset{\sim}{u}'(\tau) \rangle = 2\pi \int_k^{\infty} dk \ F/\omega^* \qquad (13a)$$

The relaxation frequency ω^* is to be determined by the very cascade process (5b) which introduced λ_k. We write such a process for the energy band Gdk, according to (5b)

$$\omega^* G\, dk = (\lambda_B + \lambda_k)\, 2k^2 G\, dk$$

or simply

$$\omega^* \cong 2\lambda_k k^2 \qquad (13b)$$

as

$$\lambda_k \gg \lambda_B$$

in the spectral region dominated by λ_k.

We rewrite the system (13), with the consideration of (10b), as follows:

$$\lambda_k = \frac{\pi}{3} \int_k^{\infty} dk \ (F/k^2 \lambda_k)$$

yielding the solution

$$\lambda_k = (4\pi/3)^{\frac{1}{2}} \left(\int_k^\infty dk \ F k^{-2}\right)^{\frac{1}{2}} \tag{14}$$

The formulas (12) and (14) are the important expressions of transport coefficients needed in the determination of the spectra.

5. ENERGY SPECTRA

With the use of (12b), (12d) and (12e), we can reduce (8) to the system

$$(\nu + \nu_k) R^\circ + \lambda_k J^\circ = \varepsilon, \qquad \varepsilon = \nu R \tag{15a}$$

$$(\lambda_B + \lambda_k) J^\circ = \eta, \qquad \eta = \lambda_B J \tag{15b}$$

determining the flow of energy across and between the spectra. Each spectrum F or G possesses a source, a sink and a nonlinear transfer across the spectrum. The source maintains the high Reynolds number of turbulence, in such a way that its structure does not appear directly in the universal range of the spectrum (range of large k), but its amount has to balance the total rate of energy dissipation which was written on the right hand sides of respective equations (15a) and (15b). The sink is a dissipation of thermal origin: it is νR° proportional to the molecular viscosity due to collisions in the F-spectrum, and is $\lambda_B J^\circ$ proportional to the Bohm diffusion in the G-spectrum. It is to be remarked that the Bohm diffusion can also be considered as of thermal nature, since λ_B is proportional to $a^2 = KT_e/M$, according to (12c). Hence the smallest scale of the G-spectrum is the Larmor radius a/ω_c or λ_B/a. Across each individual spectrum, there exists an energy transfer characteristic of the nonlinear interactions. This transfer is $\nu_k R^\circ$ in the F-spectrum and $\lambda_k J^\circ$ in the G-spectrum. This description completes the energy flow in the G-spectrum, as indicated by (15b). However, the transfer of electrostatic energy has to be driven by the kinetic energy, requiring a coupling term of equal value

$$\Phi_k = \lambda_k J^\circ$$

see (12e). This coupling is an additional loss to the F-spectrum. In this way, we have completed the flow of energy in the F-spectrum, as indicated by (15a).

6. SPECTRUM IN A COLLISIONLESS PLASMA

The equations (15) represent a system of rather complicated and comprehensive dynamical processes. Simplifications are necessary for their solutions. To this end, consider first the case where the collision is negligible. The F-spectrum transfers its energy across the spectrum by an amount $\nu_k R^\circ$, to be converted into electrostatic energy. The latter perpetuates its full pattern (15b). In order to assure this flow effectively, the F-spectrum, which plays the role of a driving force, should have a high Reynolds number, and therefore should be in the inertial range, while the G-spectrum, having a low Reynolds number, is in the dissipative range, in view of the dominant Bohm diffusion causing an early drop of the G-spectrum.

The interactions between an inertial F-spectrum and a dissipative G-spectrum can be simply described by a differential form of (15), written as follows:

$$\begin{aligned} \nu_k \frac{dR^\circ}{dk} - \lambda_B \frac{dJ^\circ}{dk} &= 0 \\ \frac{d\lambda_k}{dk} J - \lambda_B \frac{dJ^\circ}{dk} &= 0 \end{aligned} \tag{16}$$

and obtained by the approximations

$$R^\circ \cong 0, \qquad J^\circ \cong J, \qquad \lambda_k \ll \lambda_B$$

An addition of the two equations (16) gives

$$\nu_k \frac{dR^\circ}{dk} = - \frac{d\lambda_k}{dk} J$$

or, with the use of (12a) and (14),

$$2(\pi/3)^{\frac{1}{2}}\, k^4 \Big(\int_k^\infty dk\, F\Big)\Big(\int_k^\infty dk\, F k^{-2}\Big)^{\frac{1}{2}} = \omega_c J \equiv \omega_o^3$$

yielding the solution

$$F = (12/\pi)^{1/3}\, \omega_o^2\, k^{-3} \tag{17a}$$

in the inertial subrange. It entails from (14) and (15b)

$$G = (2\pi/3)^{1/3}\, \eta\, \omega_o\, \lambda_B^{-2}\, k^{-5} \tag{17b}$$

in the dissipative subrange.

The spectral law k^{-5} , as predicted by (17b), has been verified by experiments.[4] Dimensional arguments based on the parameters a and ω_c have proposed earlier a spectral law, for provisional experimental usage,

$$G = \mathrm{const}\; a^{-2}\, \omega_c^4\, k^{-5} \qquad (17c)$$

Since a spectrum should have its magnitude dependent on the rate of dissipation, the proposed formula (17c) cannot be fully valid.

7. SPECTRUM IN A COLLISIONAL PLASMA

The collision is assumed to dominate the coupling term Φ_k, decoupling the two energy equations (15a) and (15b). The F-spectrum in the inertial subrange is determined by (15a), reduced to

$$\nu_k R^\circ = \varepsilon$$

giving the solution

$$F = (3/2\pi)^{\frac{1}{2}} (\varepsilon \omega_c)^{\frac{1}{2}} k^{-2} \qquad (18)$$

Under the inertial regime (18) of the F-spectrum, the G-spectrum is determined by (15b), rewritten in the form

$$J^\circ = \eta\, (\lambda_B + \lambda_k)^{-1}$$

or, after integration and use of (14).

$$G = \frac{3\beta}{4}\, \eta\, (\varepsilon \omega_c)^{-1/4} k^{-3/2} \left[1 + \beta\, (k/k_B)^{3/2}\right]^{-2} \qquad (19)$$

where

$$\beta = (27/8\pi)^{1/4}, \qquad k_B = (\varepsilon\, \omega_c / \lambda_B^4)^{1/6}$$

The formula (19) for the G-spectrum reduces to its inertial law for $k \ll k_B$,

$$G = \frac{3\beta}{4}\, \eta\, (\varepsilon \omega_c)^{-1/4} k^{-3/2} \qquad (20a)$$

and to its dissipative law, for $k \gg k_B$,

$$G = \frac{3}{4\beta}\, (\varepsilon \omega_c)^{1/4} \lambda_B^{-2}\, \eta\, k^{-9/2} \qquad (20b)$$

The transitions from the inertial law (20a) to the dissipative law (20b) occurs at the critical wave-number $\beta^{-2/3} k_B$.

8. CONCLUSIONS

By means of a hydrodynamic model, valid for a plasma of hot electrons and cold ions, we have established the development of spectra of kinetic and electrostatic energies. The nonlinear interactions have been described by two transfer functions valid for the nonlinear transfer of modes across each spectrum, and by a production function serving the coupling between the two spectra. The closure problem, as characteristic to any nonlinear system, has of course to be solved, and in the present theory, the approximation is resorted to the cascade concept. The concept helped in the derivation of the equations for the energy spectra, and in the formulation of a "turbulent relaxation frequency". The latter frequency is very important in determining the structure of turbulent transport coefficients entering in all the above nonlinear transport phenomena. Finally the equations of energy spectra are solved for the collisonal and collisionless cases. Some of the theoretical results have been confirmed by experiments in gas plasmas. It seems that experiments in solid plasmas, which often are simpler, should soon be able to check the theoretical predictions more fully.

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